Chapter 50
Research on Fast Satellite Selection Algorithm Based on Geometry

Pengfei Zhang, Chengdong Xu, Chunsheng Hu and Ye Chen

Abstract The positioning accuracy with Global Navigation Satellite System (GNSS) depends on both the pseudorange error and the geometric dilution of precision (GDOP), and GDOP is depended on the geometry of user/satellites which are selected. Positioning with a GNSS needs at least four satellites which are visible for the user so that at least four pseudorange measurements can be provided to determine the user position in three dimensions and the receiver clock offset. In order to select as few as visible satellites whose geometry is better, a new fast satellite selection algorithm based on geometry is come up with by this paper through analyzing geometry characteristics of the optimal visible satellites combination. The expected visible satellites are selected based on the distribution characteristics of elevation angles and azimuth angles combined with tetrahedron volume. Taking GPS, GLONASS and GALILEO system for examples, this paper compares the algorithm proposed by this paper with the best geometry satellite selection algorithm through simulation. The simulation results show that the algorithm proposed by this paper solves the problem that there are a lot of matrix multiplications and matrix inversions in the best geometry satellite selection algorithm. So the new algorithm can reduce computational complexity and increase receiver processing speed. The theory of the algorithm is simple and easy to use. Meanwhile, the algorithm can satisfy the real time requirements for users.

Keywords GNSS · GDOP · Geometry · Satellite selection algorithm

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50.1 Introduction

The four Global Navigation Satellite Systems (GNSSs) which are currently on-orbit operation include the American GPS, the Russian GLONASS, the European GALILEO system and the Chinese BD2 system (the Second Generation of BeiDou Navigation System) [1]. The positioning accuracy with GNSS mainly depends on pseudorange error and geometric dilution of precision (GDOP), and GDOP is depended on user/satellites geometry [2]. In order to determine user position in three dimensions and the receiver clock offset, at least four satellites should be visible to provide four pseudorange measurements [3]. So it is important that how to select the visible satellites with better geometry and less quantity fast. The best geometry satellite selection algorithm is described simply as follows: find all combinations with four satellites in all visible satellites and calculate GDOP or tetrahedron volume in all combinations firstly, and then select the combination with minimal GDOP or maximal tetrahedron volume as final satellite selection result. The result of best geometry satellite selection algorithm is optimal and the corresponding positioning accuracy is best. However, it has a heavy calculation burden and it is time-consuming. Especially for the high-dynamic user, it has a disadvantage in real time performance. Therefore, it is significant to find out a fast satellite selection algorithm with less calculation on the premise that it has little influence on positioning accuracy. This paper analyzes the geometry characteristics of the optimal combination which is selected by best geometry satellite selection algorithm and summarizes the distribution regularities of elevation angles and azimuth angles in the combination. Then a new fast satellite selection algorithm based on geometry is come up with. At last, taking GPS, GLONASS and GALILEO system as examples, this paper compares the algorithm proposed by this paper with the best geometry satellite selection algorithm trough simulation.

50.2 Best Geometry Satellite Selection Algorithm

In GNSS, the positioning error can be expressed as a product of GDOP and pseudorange error [4].

$$\sigma_p = GDOP \times \sigma_{UERE} \tag{50.1}$$

where σ_p is the positioning error and σ_{UERE} is the pseudorange error. As is shown in formula (50.1), GDOP is a linear mapping from peseudorange error to positioning error. On the condition of the same pseudorange error, the less the GDOP, the less the positioning error. Therefore, a basic principle of satellite selection is to make GDOP of the selected combination as small as possible.

If there are $n(n \geq 4)$ simultaneous and continuous satellites of one GNSS in view of users in one area at one moment, a best geometry satellite selection algorithm is to go through the entire four satellites combinations in all visible satellites and select the combination with minimal GDOP as a final satellite selection result. GDOP is inversely proportional to tetrahedron volume which consists of the end points of unit vectors between a user and satellites. So another best geometry satellite selection algorithm is to select the combination with maximal tetrahedron volume as a final satellite selection result [5]. Assuming that positions of the selected four satellites in Earth-Centered Earth-Fixed (ECEF) coordinate system are $(x_i, y_i, z_i)\,(i = 1, 2, 3, 4)$, the position of user in ECEF coordinate system is (x_u, y_u, z_u), the calculation formula of GDOP is as follows [6]:

$$GDOP = \sqrt{trace(H^T H)^{-1}} \tag{50.2}$$

Where,

$$H = \begin{bmatrix} a_{x1} & a_{y1} & a_{z1} & 1 \\ a_{x2} & a_{y2} & a_{z2} & 1 \\ a_{x3} & a_{y3} & a_{z3} & 1 \\ a_{x4} & a_{y4} & a_{z4} & 1 \end{bmatrix}$$

$$a_{xi} = \frac{x_i - x_u}{r_i},\ a_{yi} = \frac{y_i - y_u}{r_i},\ a_{zi} = \frac{z_i - z_u}{r_i}$$

$$r_i = \sqrt{(x_i - x_u)^2 + (y_i - y_u)^2 + (z_i - z_u)^2} \quad (i = 1, 2, 3, 4)$$

The calculation formula of tetrahedron volume V is as follows [7]:

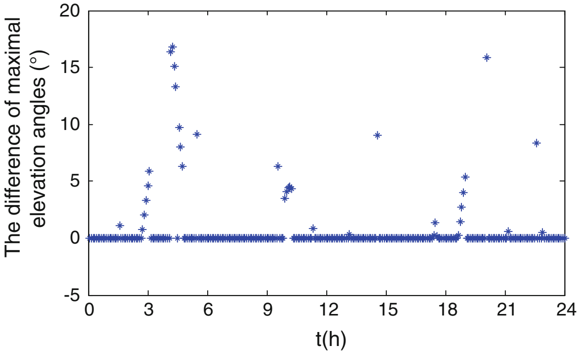
$$V = \frac{1}{6}|H| \tag{50.3}$$

There are C_n^4 times calculation of GDOP or tetrahedron volume in best geometry satellite selection algorithm. Taking GPS as an example, there are 6–12 simultaneous and continuous satellites in view of a user in one area at one moment. That is, there are 15–495 times calculation of GDOP or tetrahedron volume in best geometry satellite selection algorithm. As is shown in formula (50.2) and formula (50.3), it involves matrix multiplication, matrix inversion or determinant computation in every time of calculation, the computation burden is heavy and it is time-consuming.

50.3 Analysis of Best Geometry Characteristics

Taking GPS as an example, the reference time of ephemeris and start time of simulation are set as 0 h 0 min 0 s January 1, 2012 UTC, the total time of simulation is set as 24 h. The observation site is set as 39°N and 116°E and the mask angle is set as 5°. The satellite selection result of best geometry satellite selection

Fig. 50.1 The difference of maximal elevation angles between satellite selection result and all the visible satellites

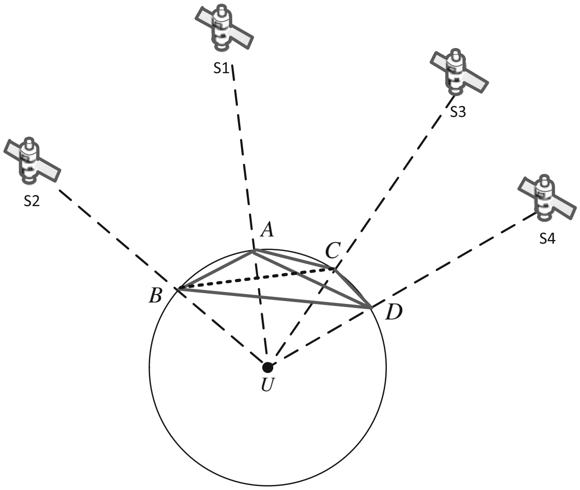


algorithm is analyzed and the difference between maximal elevation angles of the satellite selection result and that of all the visible satellites is shown statistically in Fig. 50.1.

As is shown in Fig. 50.1, there is nearly 90 % simulation time that the satellite with maximal elevation angle in satellite selection result and the satellite with maximal elevation angle in all the visible satellites are the same, and there is more than 95 % simulation time that the difference of maximal elevation angle between them is less than 10°. It indicates that there should be a satellite with greater elevation angle in best geometry. Next, the geometry characteristics of the other three satellites are analyzed according to vertex distribution of tetrahedron. The tetrahedron which consists of the end points of unit vectors between a user and satellites is shown in Fig. 50.2.

In Fig. 50.2, the tetrahedron ABCD is consisted of four projective points of the four satellites located on the unit sphere whose center is the user [8]. The size of

Fig. 50.2 Tetrahedron geometry of four satellites



tetrahedron volume reflects the quality of satellite selection result. The larger the volume, the better the result. In order to make the tetrahedron volume as large as possible, when a satellite with a greater elevation angle is determined, other three satellites should be selected as follows: the elevation angles of the three satellites should be small and the difference between that of each other should be also small, then the azimuth angles of them should be distributed uniformly.

50.4 Fast Satellite Selection Algorithm Based on Geometry

Through analyzing the geometry characteristics of the optimal visible satellites combination, the expected visible satellites are selected based on the distribution characteristics of elevation angles and azimuth angles combined with tetrahedron volume. The selection result is not optimal, but it can increase the satellite selection speed with less calculation on the premise that it has little influence on positioning accuracy. The detailed satellite selection process is shown in Fig. 50.3.

Step 1 Set the mask angle according to the field of vision to the satellites in the location of the receiver. The mask angle is inversely proportional to the field of vision. The wider the field of vision, the lower the mask angle should be set.

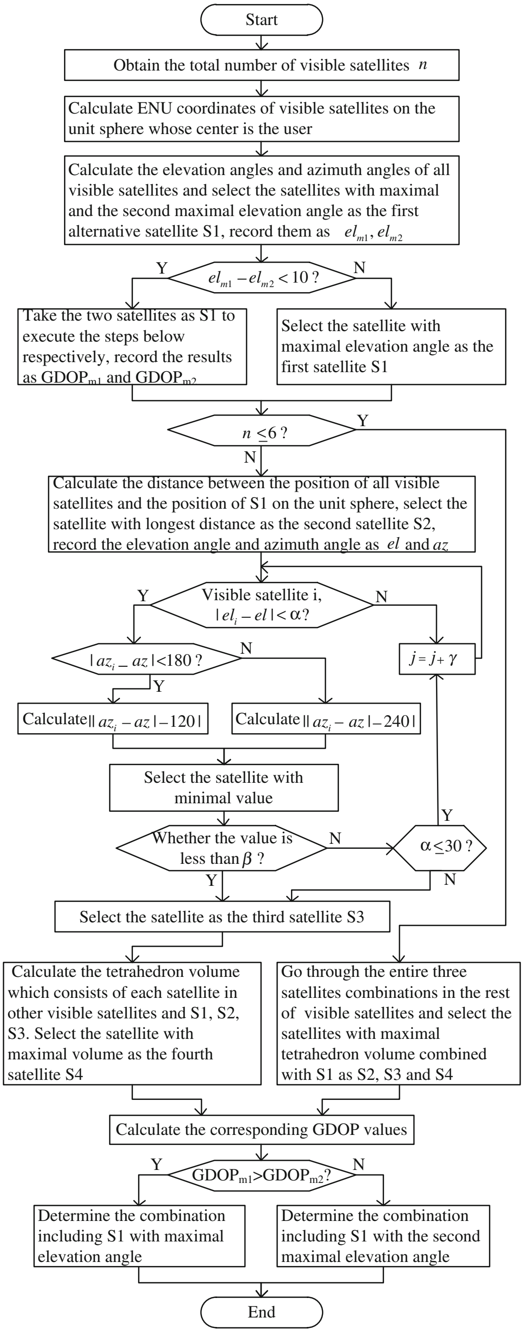
Step 2 Eliminate the unhealthy satellites in GNSS according to the parameters which represent the health state in ephemeris, calculate the position coordinates of the health satellites in ECEF coordinate system and get the total number of the visible satellites n according to the mask angle set in step 1.

Step 3 Transform the visible satellites coordinates in ECEF coordinate system to ENU coordinate system (the origin is the user, X-axis points to east orientation, Y-axis points to north orientation and Z-axis points to zenith orientation) and get the unit vector [9].

Step 4 Calculate the elevation angles and azimuth angles of all visible satellites in ENU coordinate system and select the satellites with maximal and the second maximal elevation angle as the first alternative satellite S1 in satellite selection result. Judge the difference between elevation angles of the two satellites, if it is less than 10°, select two combinations according step 5–step 8, if not, determine the satellite with maximal elevation angle as the first satellite S1 in satellite selection result.

Step 5 If $n \leq 6$, go through the entire three satellites combinations in the rest of visible satellites and select the satellites with maximal tetrahedron volume combined with S1 as the second S2, the third S3 and the fourth satellite S4 in satellite selection result respectively, and then turn to step 9; if $n > 6$, turn to step 6.

Fig. 50.3 Process of satellite selection



Step 6 Calculate the distance between the position of all visible satellites and the position of S1 on the unit sphere, select the satellite with longest distance as the second satellite S2 in satellite selection algorithm.

Step 7 Select the third satellite S3.

Step 7.1 Calculate the difference between elevation angle of S2 and that of other visible satellites except S1 and get the satellites whose absolute values of difference are less than α. In order to make the tetrahedron volume as large as possible, the elevation angles of S3 and S2 should be in the same range, that is, the smaller α, the better the result. In general, α is less than 30°;

Step 7.2 Calculate the difference between azimuth angle of S2 and that of other visible satellites obtained in step 7.1. If absolute value of the difference is greater than 180°, calculate the difference between the absolute value and 240°; if not, calculate the difference between the absolute value and 120°;

Step 7.3 Find out the minimal absolute value in the difference calculated in step 7.2, judge whether the minimal value is less than β (the size of β reflects azimuth angles distribution of S3 and S2, the smaller β, the more uniform azimuth angles distribution of S3 and S2, the larger the tetrahedron volume). If it is less than β, select this satellite as the third satellite S3 in satellite selection result. If not, increase the value of α by a certain step and repeat step 7.1–7.2 until find out the third satellite. If the third satellite met the conditions is not selected when the value of α increases to 30°, select the satellite with minimal β when α is 30° as the third satellite S3 in satellite selection result.

Step 8 Calculate the tetrahedron volume which consists of each satellite in other visible satellites and S1, S2, S3. Select the satellite with maximal volume as the fourth satellite S4 in satellite selection result.

Step 9 If there are two schemes in step 4 about S1, judge the GDOP size of the two schemes. Select the combination with the minimal GDOP as the satellite selection result.

50.5 Simulation Analysis

In the simulation, 32 GPS satellites (according to Yuma almanac), 24 GLONASS satellites (uniformly locate in 3 orbital planes 120° apart in right ascension) and 27 GALILEO satellites (3 orbital planes, equally spaced and with 56° nominal inclination and 9 satellites per plane) are chosen respectively [10]. The reference time of ephemeris and start time of simulation are set as 0 h 0 min 0 s January 1, 2012 UTC, the total time of simulation is set as 24 h. The observation site is set as 39°N and 116°E and the mask angle is set as 5°. Both α and β are set as 30°, γ is set

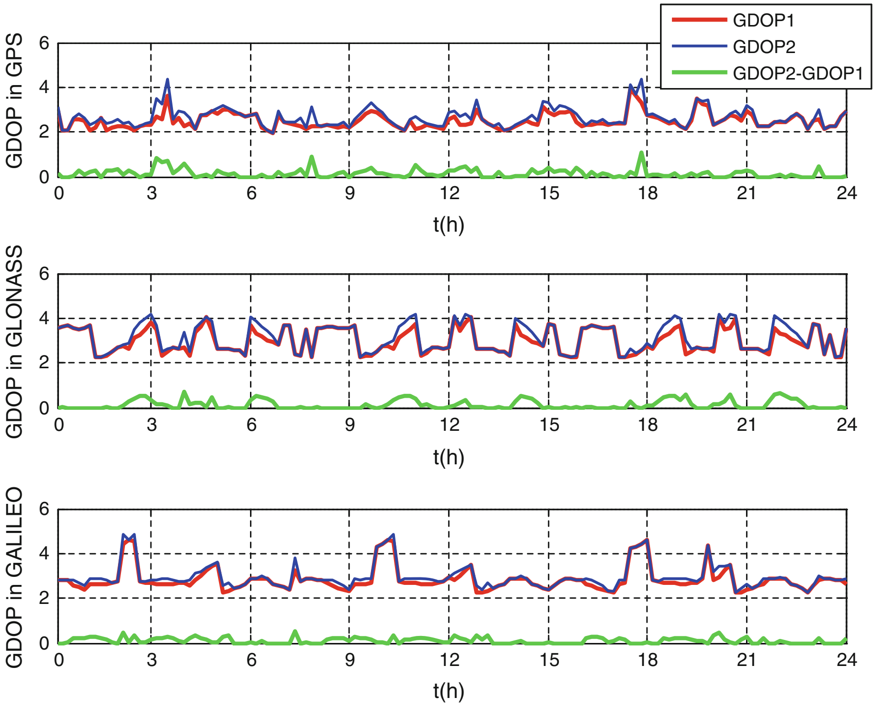


Fig. 50.4 Comparison of the corresponding GDOP values in the two algorithms

as 5°. The satellite selection results of best geometry satellite selection algorithm and fast satellite selection algorithm based on geometry proposed by this paper are simulated. The corresponding GDOP and consuming time are compared respectively. The comparison results are shown in Figs. 50.4 and 50.5.

In Figs. 50.4 and 50.5, GDOP1 and T1 represent the GDOP value and consuming time of satellite selection result in best geometry satellite selection algorithm respectively. GDOP2 and T2 represent the GDOP value and consuming time of satellite selection result in fast satellite selection algorithm based on geometry proposed by this paper respectively. The mean values of the difference between GDOP1 and GDOP2 in GPS, GLONASS and GALILEO system are 0.1470, 0.1377 and 0.1097 respectively. The mean increase of corresponding positioning error is less than 5 %. So it is an acceptable influence on positioning in fast satellite selection algorithm based on geometry. In addition, the calculation time is reduced obviously and the speed of satellite selection is increased about ten times. Therefore, it achieves the purpose that as few as visible satellites are selected fast with less calculation on the premise that it has little influence on positioning accuracy.

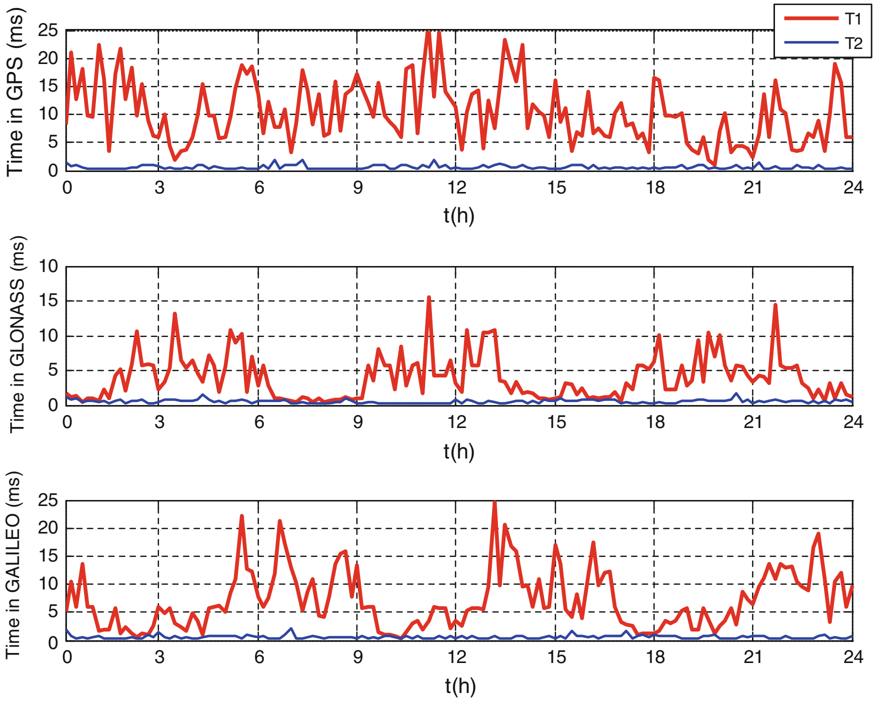


Fig. 50.5 Comparison of satellite selection time in the two algorithms

50.6 Conclusion

This paper analyzes the geometry characteristics of the optimal combination which is selected by best geometry satellite selection algorithm and summarizes the distribution regularities of elevation angles and azimuth angles in the combination. The selection result of the best geometry satellite selection algorithm is optimal, but it has a heavy calculation burden and it is time-consuming. Therefore, a fast satellite selection algorithm based on geometry is come up with by this paper. In this algorithm, the expected visible satellites are selected based on the distribution characteristics of elevation angles and azimuth angles combined with tetrahedron volume. At last, taking GPS, GLONASS and GALILEO system as examples, this paper compares the algorithm proposed by this paper and best geometry satellite selection algorithm trough simulation. The simulation results show that the algorithm proposed by this paper solves the problem that there are a lot of matrix multiplications and matrix inversions in the best geometry satellite selection algorithm so that it can satisfy the real time requirements for users. The algorithm proposed by this paper can be used in any one of the GNSSs and it can also be expanded to the satellite selection of multi-GNSSs.

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